



# *Trees*

Prof. Harish D.G.

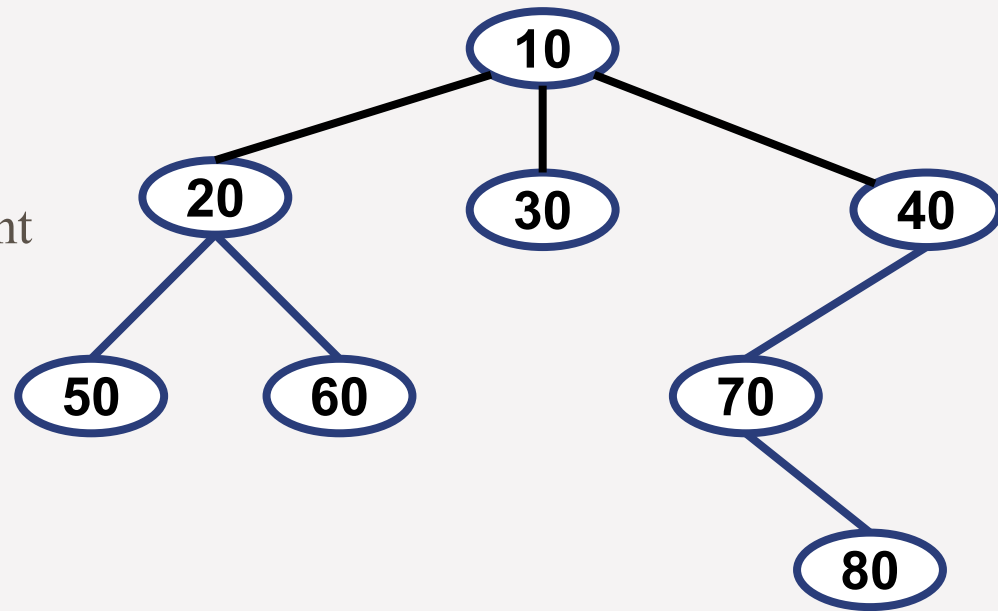
Dept. of Computer and IT

College of Engineering, Pune (COEP)

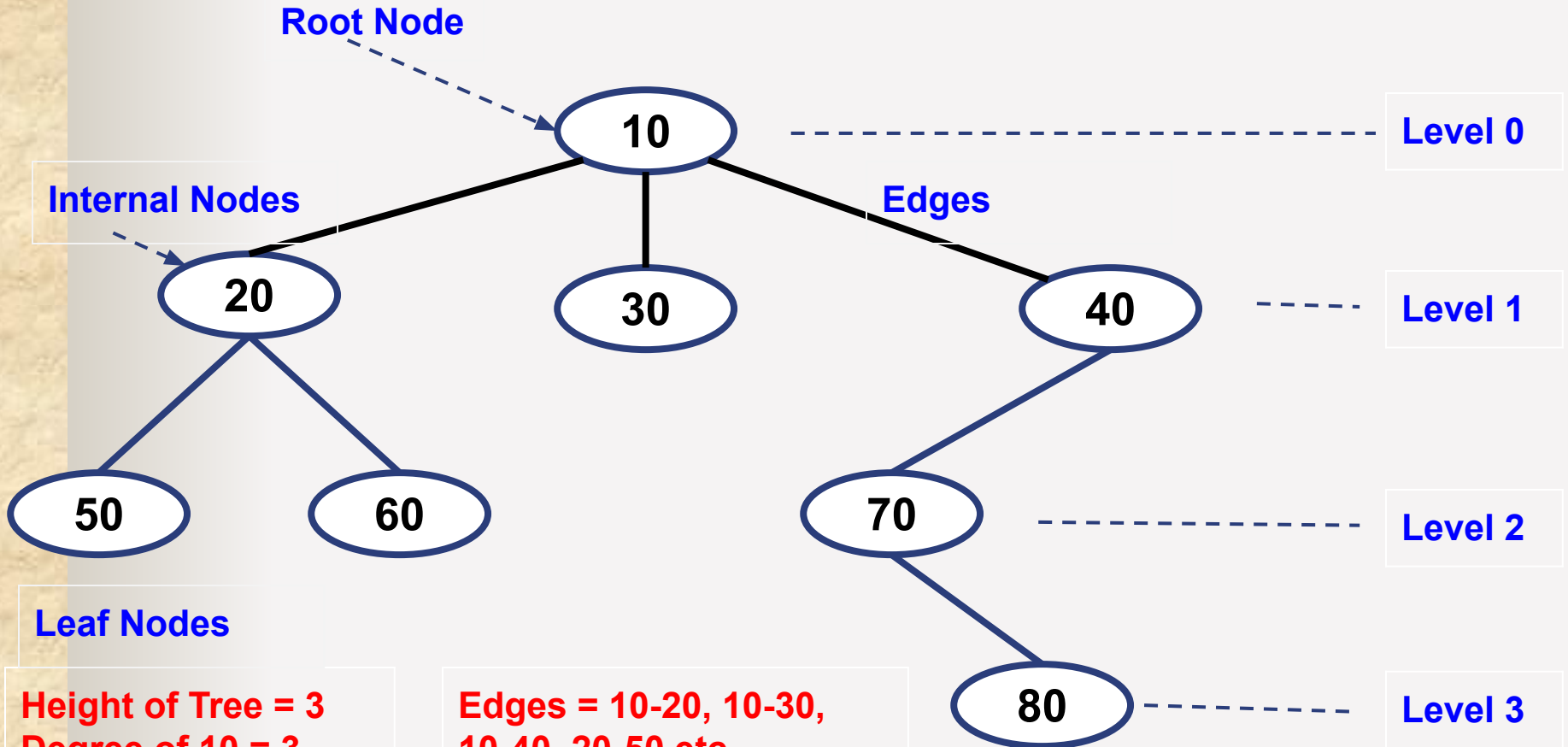
[www.harishgadade.com](http://www.harishgadade.com)

# Tree

1. Introduction
2. Basic terms/Terminology
  1. Definition of Tree
  2. Node
  3. Degree
  4. Leaf Nodes
  5. Interior Nodes
  6. Children and Parent
  7. Siblings
  8. Degree of tree
  9. Level
  10. Height of tree



# Tree



Height of Tree = 3  
Degree of 10 = 3  
Degree of 40 = 1  
Degree of Tree = 3

Edges = 10-20, 10-30, 10-40, 20-50 etc  
Height of Tree = 3

Internal Nodes = 20, 40, 70  
Leaf Nodes = 50, 60, 30, 80

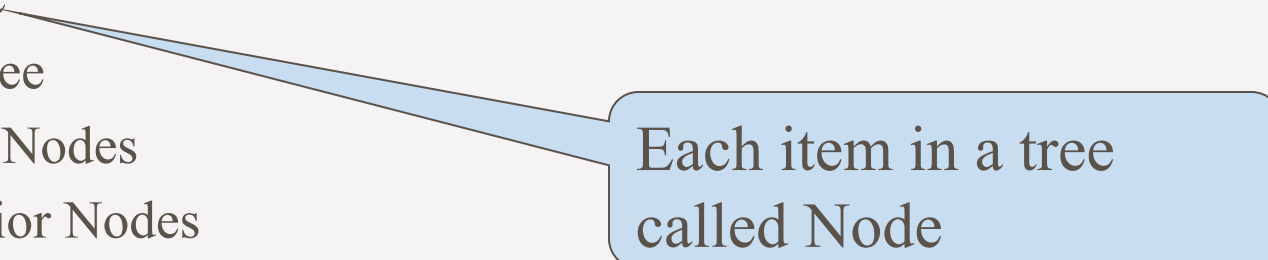
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A tree is a DS with set of one or more nodes. There is a special node called root and remaining nodes are partitioned into disjoint groups  $T_1, T_2, \dots, T_n$ .  
Where  $T_i$ - sub tree

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Each item in a tree called Node

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Number of subtrees of a node in a given tree

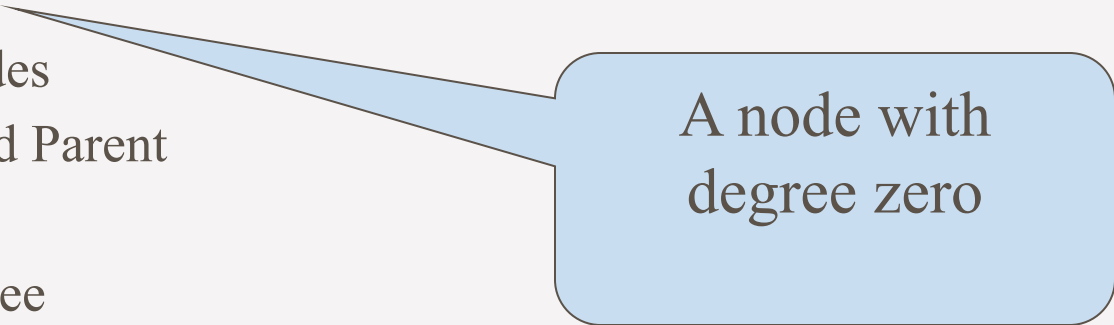
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Maximum degree of node in a given tree

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A node with  
degree zero



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Any Node except root node whose degree not zero

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Root of sub tree is parent & sub tree is its children

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Children's of the same parents

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Maximum degree  
of nodes in a given  
tree

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Root is at level 0  
If node is at level 'n',  
then its children's are  
at level  $n+1$



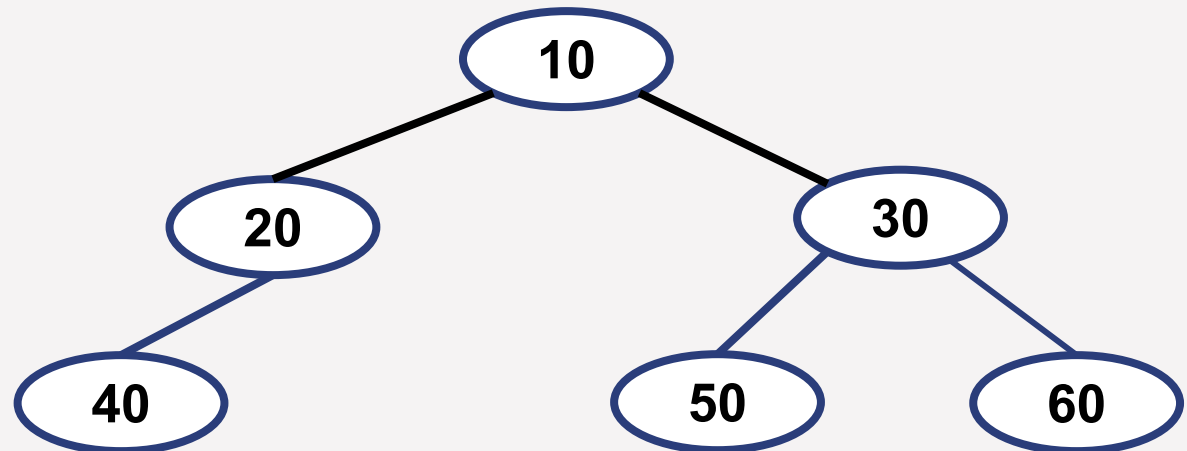
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Maximum level of any node in a given tree

# Binary Tree

- ❑ Definition
- ❑ Properties of Binary tree
- ❑ Types of Binary Tree
  1. Skewed Binary tree
  2. Complete Binary Tree
  3. Full Binary Tree



# Binary Tree

- ❑ Definition
- ❑ Properties of Binary tree
- ❑ Types of Binary Tree
  1. Skewed Binary tree
  2. Complete Binary Tree
  3. Full Binary Tree

## Properties of BT:

1. For Binary Tree, maximum number of nodes at level L are  $2^L$
2. A full Binary tree of height 'h' has  $(2^{h+1} - 1)$  total nodes
3. Total number of external nodes in a binary tree are internal nodes + 1.

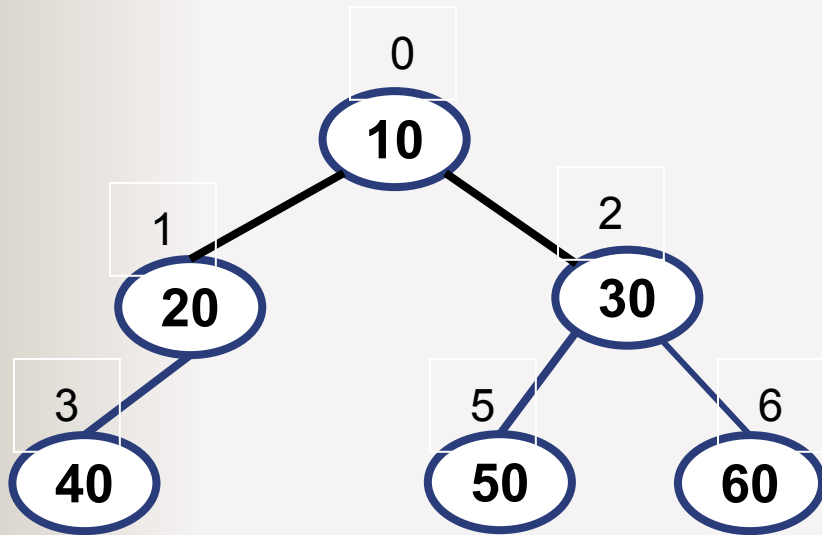
**i.e.  $E = I + 1$**

## Rules:

- $\text{Parent}(i) = \text{floor}(i-1)/2$
- $\text{Leftchild}(i) = (2i+1)$
- $\text{Rightchild}(i) = (2i + 2)$



# Binary Tree



$$\text{Level } 0 = 2^0 = 1$$

$$\text{Level } 1 = 2^1 = 2$$

$$\text{Level } 2 = 2^2 = 4$$

$$\begin{aligned} \text{Total Nodes} &= 2^{(h+1)} - 1 \\ &= 2^3 - 1 = 7 \end{aligned}$$

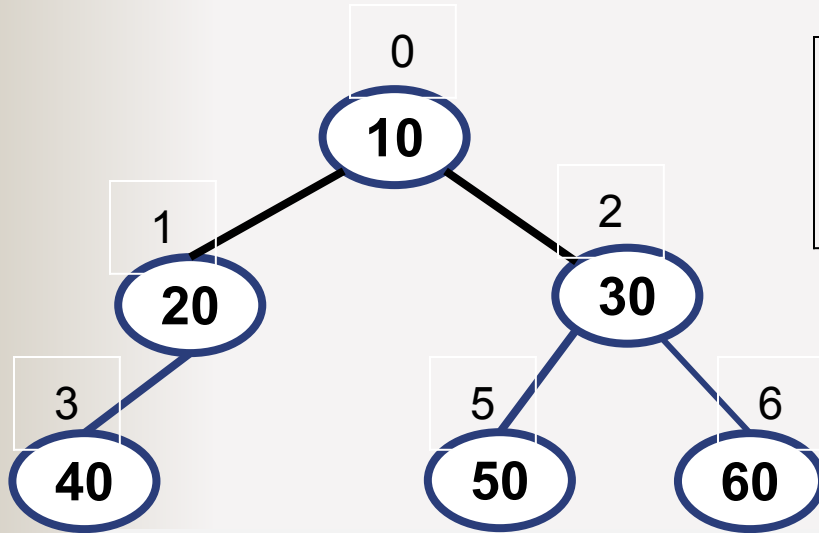
## Properties of BT:

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2. A full Binary tree of height 'h' has  $(2^{h+1} - 1)$  total nodes
3. Total number of external nodes in a binary tree are internal nodes(Including Root) + 1.  
**i.e.  $E = I + 1$**

$$E = I + 1$$

$$E = 3 + 1 = 4$$

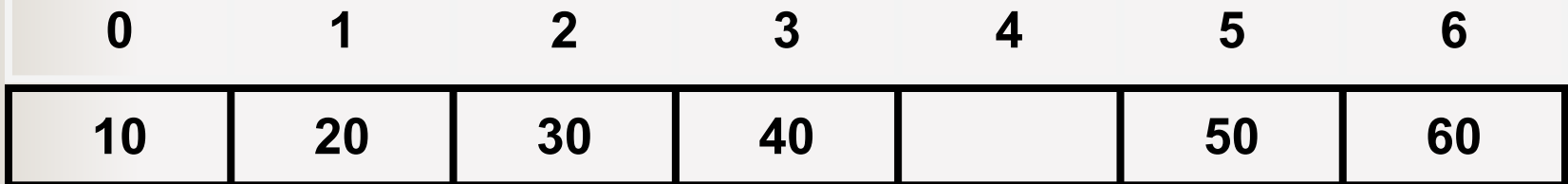
# Array Representation Binary Tree



Level 0 =  $2^0 = 1$   
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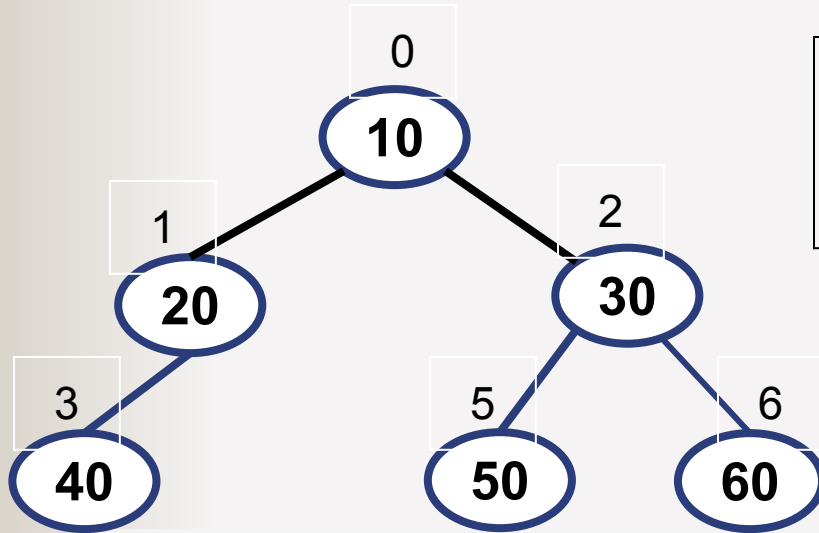


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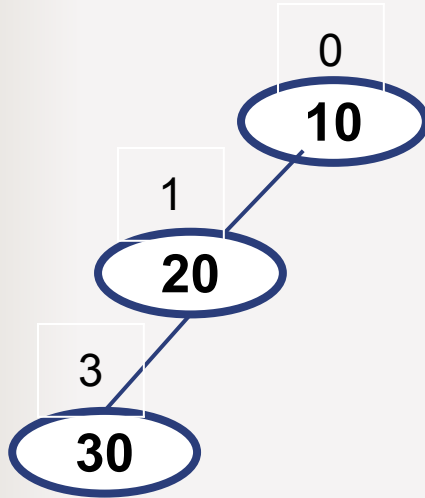
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0	1	2	3	4	5	6
10	20	30	40		50	60

Total Nodes =  $2^{(h + 1)} - 1$   
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Parent(1) = Floor(0/2) = 0  
Leftchild(1) = (2\*1+1) = 3  
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# Array Representation Binary Tree



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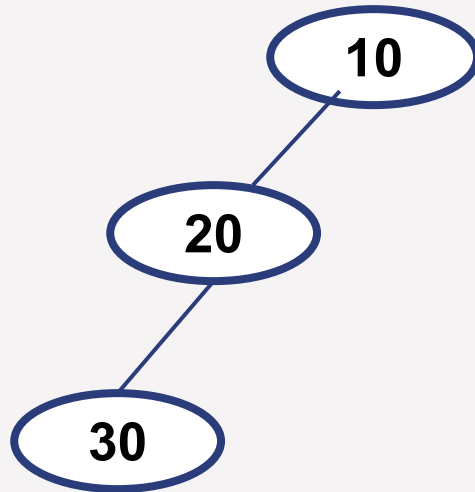
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# Binary Tree

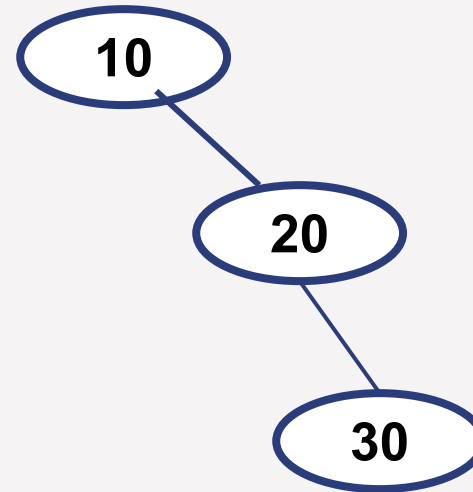
- ❑ Definition
- ❑ Properties of Binary tree
- ❑ Types of Binary Tree

## 1. Skewed Binary tree

Every node is having either only left or right subtree



**Left Skew Tree**

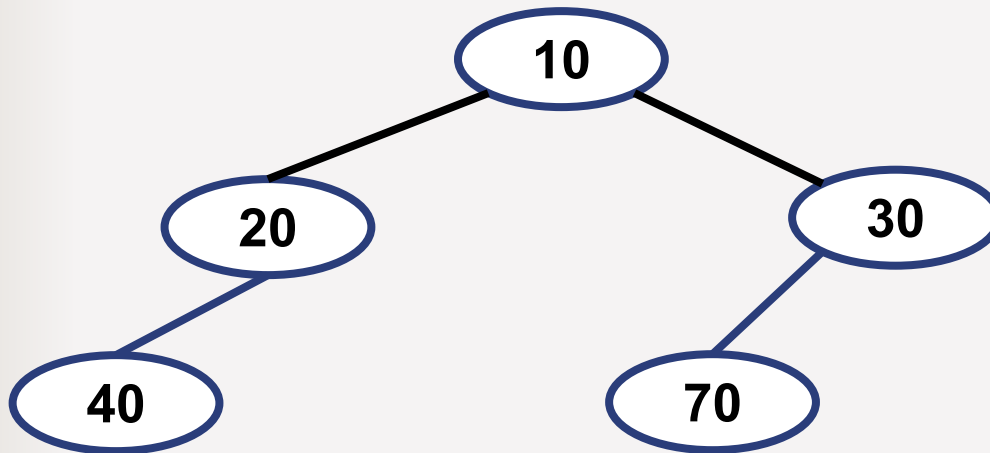


**Left Skew Tree**

# Binary Tree

- ❑ Definition
- ❑ Properties of Binary tree
- ❑ Types of Binary Tree
  1. Skewed Binary tree
  2. Complete Binary Tree

All Leaves are  
at the same  
depth / level

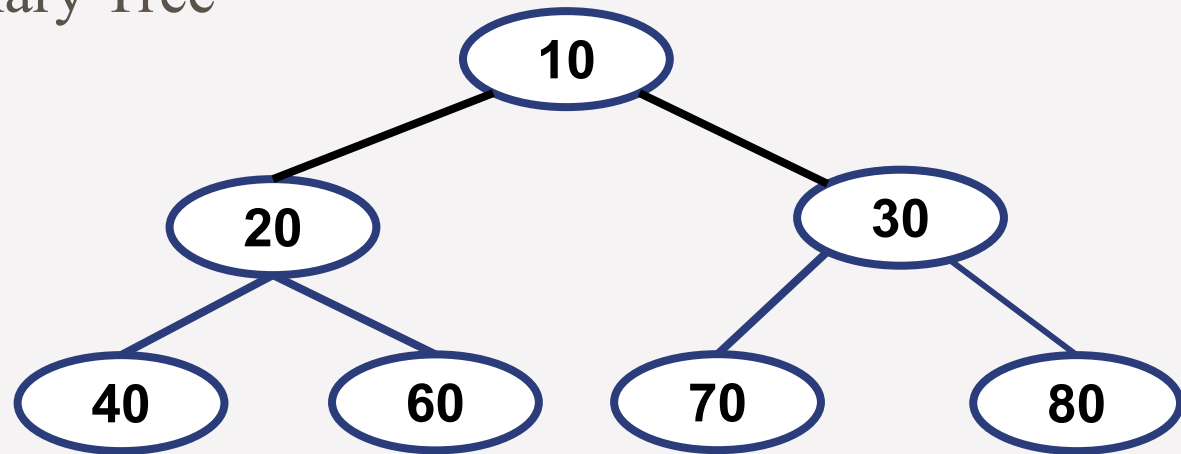


**Complete Binary Tree**

# Binary Tree

- ❑ Definition
- ❑ Properties of Binary tree
- ❑ Types of Binary Tree
  1. Skewed Binary tree
  2. Complete Binary Tree
  3. Full Binary Tree

Every node has zero or two children.  
- Also called Strictly B.T.



**Full Binary Tree**

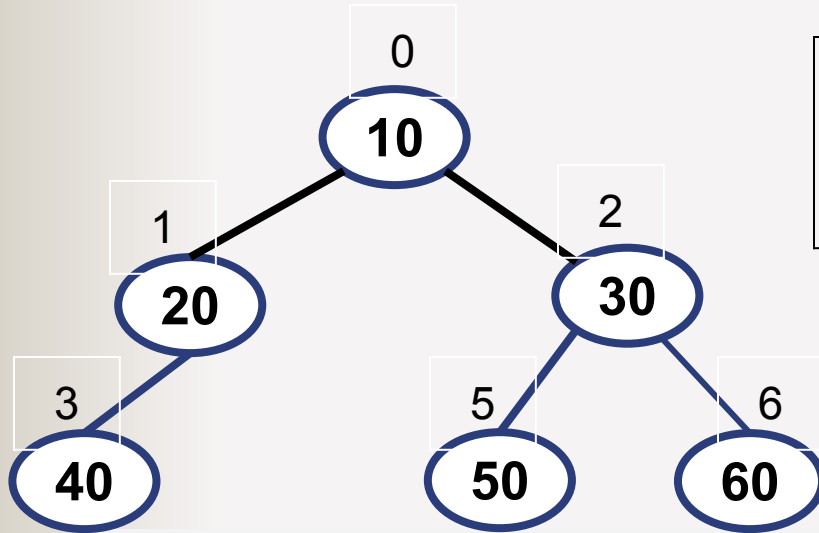


# Representation of Binary Tree

1. Sequential / Array Representation
2. Linked Representation
  - Structure Representation
  - Node Representation



# Sequential / Array Representation Binary Tree



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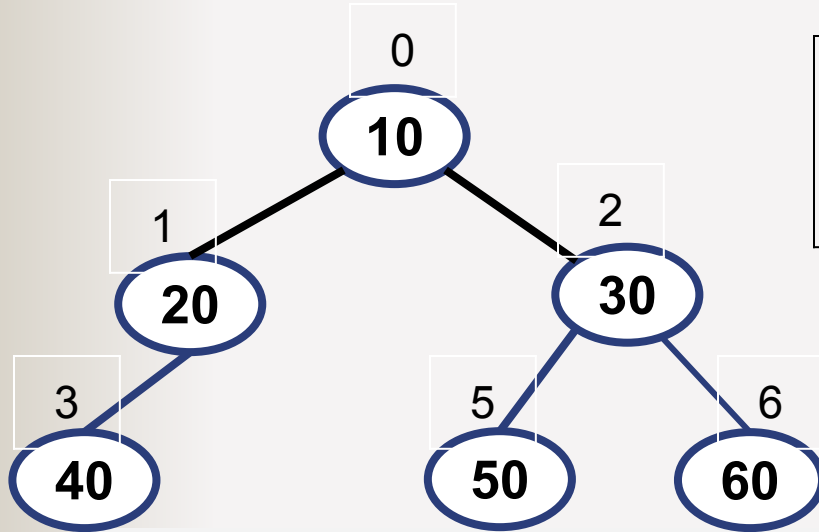
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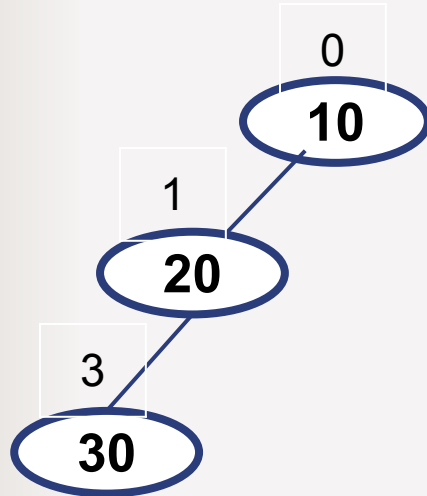
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# Linked Representation of Binary Tree

## Structure Representation:

```
struct node
{
    struct node *LC;
    int data;
    struct node *RC;
};
```



Node Structure

# Linked Representation of Binary Tree

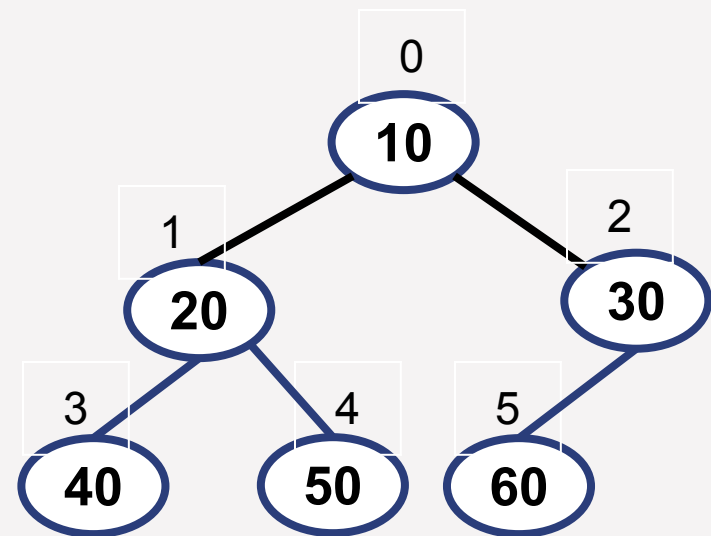
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    struct node *RC;
};
```



Node Structure

e.g. 10,20,30,40,50



# Linked Representation of Binary Tree

e.g. 10,20,30,40,50, 60

